Process control

Lecture 7

One of the most important properties of object in control theory is so-called transfer function. The transfer function of an object describes the relationship between object's output and input. It is obtained based on Laplace-transform of dynamic equations.

Consider arbitrary object presented in Fig. 1. If the object is influenced by some factor (input), as a result we will get some response (output). Both input and output can change with time, so we will denote them as $f(t)$ and $x(t)$ respectively. After applying the Laplace transform to input and output, we will obtain functions of complex variable, $F(s)$ and $X(s)$. The transfer function, typically denoted by G, combines these two functions into one property. By definition, the transfer function is the ratio of the output of a system to the input of a system, in the Laplace domain considering its initial conditions to be zero.

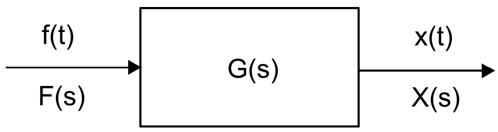

Fig. 1. Block scheme of an object

For example, let's consider the object with dynamics is described by the differential equation:

$$a\frac{dx}{dt} + bx = f(t) \tag{1}$$

After applying the Laplace transform, we get:

$$a(sX(s) + x(0)) + bX(s) = F(s) \tag{2}$$

If initial condition $x(0)$ is 0, then the transfer function of this object can be calculated according to:

$$X(s)(as+b)=F(s)\Rightarrow\frac{X(s)}{F(s)}=\frac{1}{(as+b)}\Rightarrow G(s)=\frac{1}{(as+b)} \tag{3}$$

One of advantages of using transfer functions, is that it can be easily determined for complicated systems. For example, let's take a cascade of three objects. The block scheme of such connection is presented in Fig. 2.

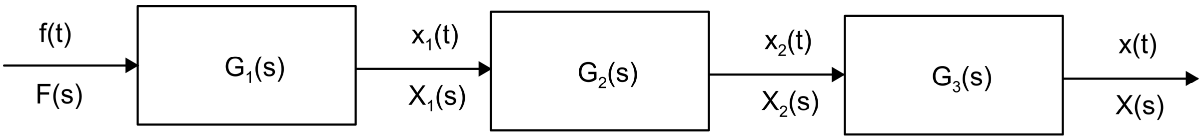

Fig. 1. Block scheme of a cascade of objects

The transfer function of the first object is:

$$G_1(s)=\frac{X_1(s)}{F(s)} \tag{4}$$

The output of the first object is an input for the second one. Therefore:

$$G_2(s)=\frac{X_2(s)}{X_1(s)} \tag{5}$$

And for the third one we have:

$$G_3(s)=\frac{X(s)}{X_2(s)} \tag{6}$$

From the Eq. (4) we take X_1(s):

$$X_1(s)=G_1(s)F(s) \tag{7}$$

and put it into Eq. (5):

$$G_2(s) = \frac{X_2(s)}{G_1(s)F(s)} \tag{8}$$

From the above equation let's take X_2(s):

$$X_2(s) = G_1(s)G_2(s)F(s) \tag{9}$$

and put it into Eq. (6):

$$G_3(s) = \frac{X(s)}{G_1(s)G_2(s)F(s)} \tag{10}$$

From the above equation we can find $X(s)/F(s)$:

$$G(s) = \frac{X(s)}{F(s)} = G_1(s)G_2(s)G_3(s) \tag{11}$$

So the transfer function of the whole system is obtained. The reasoning can be extended to a cascade of n objects:

$$G(s) = \frac{X(s)}{F(s)} = G_1(s)G_2(s)G_3(s)...G_{n-1}(s)G_n(s) = \prod_i G_i(s) \tag{12}$$

By analogous reasoning, a formula for a transfer function of other systems can be found. One of very useful type of control systems is a closed-loop control system. A block diagram of such system is presented in Fig. 3. The symbol Σ denotes the sum $F(s)+X_2$(s) if this is a positive closed-loop control system ('+' sign near the symbol), or the difference $F(s)-X_2$(s) for a negative closed-loop control system ('–' sign near the symbol). The transfer function of such connection is:

$$G(s) = \frac{G_1(s)}{1 \pm G_1(s)G_2(s)} \tag{13}$$

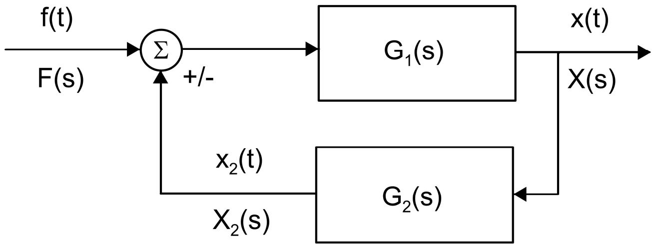

Fig. 3. Block diagram of a closed-loop system