Process control

Lecture 8

Manometer

Let's derive an equation describing the change in the liquid position in a manometer caused by gas pressure. The scheme of the manometer is presented in Fig. 1.

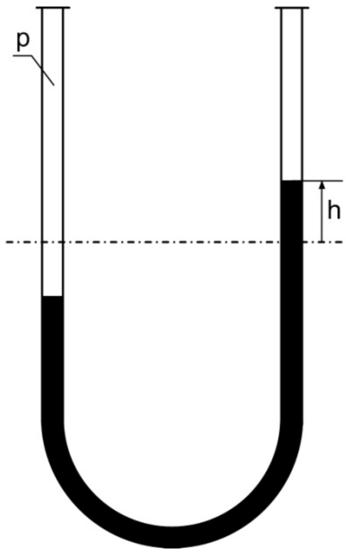

Fig. 1. Manometer scheme

Force balance for the cross-section area S and Newton's second law of dynamics is used:

$$Sp = SL\rho \frac{d^2h}{dt^2} + \frac{8SL\eta}{r_m^2}\frac{dh}{dt} + 2Sg\rho h \tag{1}$$

where:

g – gravitational acceleration

h – position of the liquid with regard to the rest position

L – length of the liquid

p – pressure

r_m – pipe radius

S – the cross-section of the manometer

t – time

ρ – liquid density

η – dynamic viscosity of the liquid

The term at the left-hand side of the above equation is related to the pressure p exerted by the gas. The first term at the right-hand side is related to the acceleration of the liquid, the second one reflects the resistances in laminar flow (see Hagen-Poiseuille equation). The last term at the right-hand side describes the hydrostatic pressure.

Equation (1) can be transformed to:

$$\frac{L}{2g}\frac{d^2h}{dt^2}+\frac{4\eta L}{r_m^2 g\rho}\frac{dh}{dt}+h=\frac{p}{2g\rho} \tag{2}$$

Let's introduce the following symbols:

$$\tau^2=\frac{L}{2g},\quad \xi=\frac{2L}{r_m^2 g\rho}\sqrt{\frac{L}{2g}},\quad f=\frac{p}{2g\rho}$$

Equation (2) will then transform to:

$$\tau^2\frac{d^2h}{dt^2}+2\tau\xi\frac{dh}{dt}+h=f \tag{3}$$

It is so-called a standard form of an equation describing a second-order object with lumped variables (a lumped object). Let's recall the definitions:

- A lumped object is one in which the dependent variables are a function of time alone. In general, this will mean solving a set of ordinary differential equations (ODEs).
- A distributed object is one in which all dependent variables are functions of time and one or more spatial variables. In this case, we will be solving partial differential equations (PDEs).

In the equation describing the manometer, the liquid level h is the dependent variable. The parameter τ in Eq. (3) is called a time constant of the object, while ξ is **damping** parameter. Function f is a driving force (please note that it incorporates the pressure, which causes the change in h). The damping parameter is important, because it decides about the way in which the objects tends to the equilibrium. For $\xi \geq 1$ the objects response is aperiodic (without **oscillations**), while for $\xi < 1$ the response is periodic, i.e. oscillatory. Figure 2 shows the response of the manometer for two values of the damping parameter. According to the definition of ξ, the oscillations can be eliminated by reducing the pipe radius, because, the damping parameter is proportional to $1/r_m^2$. One of methods is to install an orifice between manometer's arms.

The plot presented in Fig. 2 is obtained by the matlab m-file attached to the lecture. During exercises, we will solve the dynamic equation of the manometer using Laplace transform and derive the transfer function of this object.

As a homework please check units in the Eq. (2) and calculate the damping parameter for the following parameters:

L = 1.2 m, $r_m = 3.5 \cdot 10^{-3}$ m, ρ = 1000 kg/m^3, $\eta = 10^{-3}$ kg/(ms).

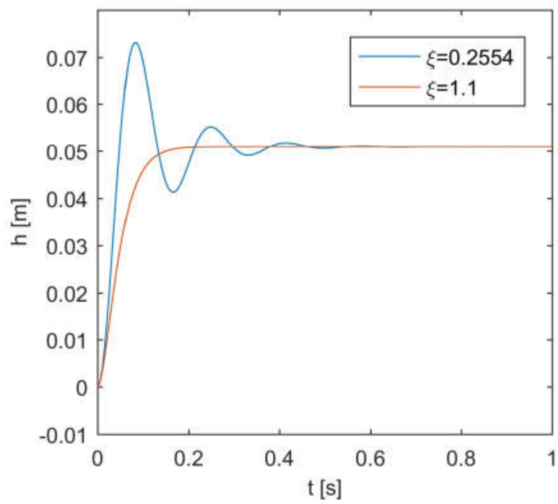

Fig. 2. Response of a manometer for two values of damping parameter